Journal of Engineering Physics and Thermophysics, Vol. 74, No. 3, 2001

NUMERICAL INVESTIGATION OF THE HYDRODYNAMICS OF A TWISTED FLOW IN A VORTEX CHAMBER BASED ON THE TWO-PARAMETER MODEL OF TURBULENCE

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UDC 532.517.4

An approach based on solution of complete averaged Navier–Stokes equations in vortex chambers using a low-Reynolds k–ε model of turbulence is considered. The problem is solved in the variables vortex, stream function, and circular component of the velocity. The method of oriented pseudoconvection is used for problems of the dynamics of twisted flows. The method allows one to retain second order of accuracy of convective terms and provide stability of the solution for rather high Reynolds numbers. The problem of formulation of boundary conditions of second order of accuracy for vorticity on a solid wall at angular points is considered. An analysis of the results obtained shows that numerical calculations within the framework of the considered model of turbulence agree with experimental data rather well.

Twisted flows possess two obvious properties – substantial predominance of inertial forces over gravitational ones and increase in the time of particle residence in vortex chambers. The structures of many devices employing twisted flows have not been changed greatly in recent times. However, problems still exist in designing new apparatus as applied to powder metallurgy and chemical processes where the separation of particles, gas scrubbing, and combustion would be efficient.

An analysis of general averaged Reynolds equations is the most promising approach to theoretical investigation of the hydrodynamics of twisted flows, although their solution involves great difficulties. These difficulties arise due to the fact that the working regions of cyclone chambers have complex geometric shapes where vortex formations, recirculation zones, and reverse flows appear in motion of a gas flow.

According to experiments, in vortex chambers the velocity of the gas is small compared to the velocity of sound, while the density and viscosity virtually do not change over the cross section; therefore an incompressible flow with constant molecular viscosity is considered. It is known that three-dimensional flows in centrifugal apparatus change to axisymmetric ones rather quickly; therefore the initial portion can be neglected. In this case, $\partial/\partial\varphi = 0$ and the system of complete averaged differential Navier–Stokes equations for a cylindrical coordinate system is solved in the variables vorticity Ω, stream function Σ, and circular component of the velocity U_φ. This approach provides identical fulfillment of the continuity equation; moreover, the number of equations investigated decreases. The equations for the variables Ω, Ψ, and U_φ can be represented in the form [1]

$$\frac{\mathrm{Re}}{2}\left[r\frac{\partial\Omega}{\partial t}+\frac{\partial\,(rU_r\Omega)}{\partial r}+\frac{\partial\,(rU_z\Omega)}{\partial z}-2\,\frac{U_\varphi}{r}\frac{\partial U_\varphi}{\partial z}\right]=$$

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 1062-0125/01/7403-0698$25.00

$$= \frac{\partial}{\partial r}\left[r\,(1+\nu_t)\,\frac{\partial \Omega}{\partial r}\right] + \frac{\partial}{\partial z}\left[r\,(1+\nu_t)\,\frac{\partial \Omega}{\partial z}\right] + f_\Omega\,,$$

$$\frac{\mathrm{Re}}{2}\left[r\,\frac{\partial U_\varphi}{\partial t} + \frac{\partial\,(rU_rU_\varphi)}{\partial r} + \frac{\partial\,(rU_zU_\varphi)}{\partial z}\right] = \tag{1}$$

$$= \frac{\partial}{\partial r}\left[r\,(1+\nu_t)\,\frac{\partial U_\varphi}{\partial r}\right] + \frac{\partial}{\partial z}\left[r\,(1+\nu_t)\,\frac{\partial U_\varphi}{\partial z}\right] - U_\varphi\left(\frac{\mathrm{Re}\,U_r}{2} + \frac{\partial \nu_t}{\partial r} + \frac{1+\nu_t}{r}\right),$$

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{r}\,\frac{\partial \Psi}{\partial r} = \Omega r^2\,,$$

with

$$f_\Omega = 2\,\frac{\partial \Omega}{\partial r}\,(1+\nu_t) + 2\,\frac{\partial^2 \nu_t}{\partial r \partial z}\left(\frac{\partial U_r}{\partial r} - \frac{\partial U_z}{\partial z}\right) + 3\Omega\,\frac{\partial \nu_t}{\partial r} +$$

$$+ \left(\frac{\partial^2 \nu_t}{\partial z^2} - \frac{\partial^2 \nu_t}{\partial r^2}\right)\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right) + r\,\frac{\partial \nu_t}{\partial z}\,\frac{\partial \Omega}{\partial z} + r\,\frac{\partial \nu_t}{\partial r}\,\frac{\partial \Omega}{\partial r}\,.$$

To close system (1) we used the two-parameter low-Reynolds k–ε model of turbulence of Jones–Launder [2]; this model consists of the equation of kinetic energy of pulsatory motion and the equation for the dissipation rate of kinetic energy

$$\frac{\mathrm{Re}}{2}\left[r\,\frac{\partial k}{\partial t} + \frac{\partial\,(rU_rk)}{\partial r} + \frac{\partial\,(rU_zk)}{\partial z}\right] = \frac{\partial}{\partial r}\left[r\left(1+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] + \frac{\partial}{\partial z}\left[r\left(1+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial z}\right] +$$

$$+ rG - \frac{2}{r}\left(\frac{2}{3}\,k + \frac{2\nu_t}{\mathrm{Re}}\,\frac{\partial U_z}{\partial z}\right) - \frac{\mathrm{Re}\,r\varepsilon}{2} - M_k\,,$$

$$\frac{\mathrm{Re}}{2}\left[r\,\frac{\partial \varepsilon}{\partial t} + \frac{\partial\,(rU_r\varepsilon)}{\partial r} + \frac{\partial\,(rU_z\varepsilon)}{\partial z}\right] = \frac{\partial}{\partial r}\left[r\left(1+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r}\right] + \frac{\partial}{\partial z}\left[r\left(1+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial z}\right] + \tag{2}$$

$$+ r\left(C_{\varepsilon 1} G\,\frac{\varepsilon}{k} - C_{\varepsilon 2}\,\frac{\mathrm{Re}}{2}\,\frac{\varepsilon^2}{k} + \frac{\mathrm{Re}}{2}\,M_\varepsilon\right),$$

where

$$M_k = 2r\left[\left(\frac{\partial k^{1/2}}{\partial r}\right)^2 + \left(\frac{\partial k^{1/2}}{\partial z}\right)^2\right];$$

$$G = \nu_t\left\{\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right)^2 + \left(\frac{\partial U_\varphi}{\partial z}\right)^2 + \left(\frac{\partial U_\varphi}{\partial r} - \frac{U_\varphi}{r}\right)^2 + 2\left[\left(\frac{\partial U_r}{\partial r}\right)^2 + \left(\frac{U_r}{r}\right)^2 + \left(\frac{\partial U_z}{\partial z}\right)^2\right]\right\};$$

$$M_\varepsilon = \frac{8\nu_t}{\mathrm{Re}^2}\left\{\left(\frac{\partial^2 U_z}{\partial z^2}\right)^2 + \left(\frac{\partial^2 U_z}{\partial r^2}\right)^2 + \left(\frac{\partial^2 U_r}{\partial z^2}\right)^2 + \left(\frac{\partial^2 U_r}{\partial r^2}\right)^2 + \left(\frac{\partial^2 U_\varphi}{\partial z^2}\right)^2 + \left(\frac{\partial^2 U_\varphi}{\partial r^2}\right)^2 + \right.$$

$$+\,2\left[\left(\frac{\partial^2 U_z}{\partial r\partial z}\right)^2+\left(\frac{\partial^2 U_r}{\partial r\partial z}\right)^2+\left(\frac{\partial^2 U_\varphi}{\partial r\partial z}\right)^2\right]\Bigg\}\,;$$

$$\nu_t = C_\mu R_t\,;\quad R_t = \frac{\text{Re}}{2}\frac{k^2}{\varepsilon}\,;\quad C_\mu = 0.09\exp\left[-3.4/(1+R_t/50)^2\right]\,;$$

$$C_{\varepsilon 1} = 1.45\,;\quad C_{\varepsilon 2} = 1.92\left[1-0.3\exp\left(-R_t^2\right)\right]\,;\quad \sigma_k = 1.0\,;\quad \sigma_\varepsilon = 1.3\,.$$

The boundary condition on the solid wall for vorticity can be determined from the Thom condition of first order of accuracy [3]

$$\Omega_{i,\text{wz}} = \frac{2\,(\Psi_{i,\text{wz}+1}-\Psi_{i,\text{wz}})}{r^2\Delta z^2}\,,\quad \Omega_{\text{w}r,j} = \frac{2\,(\Psi_{\text{w}r+1,j}-\Psi_{\text{w}r,j})}{r^2\Delta r^2}\,,$$

and also from the Woods condition of second order of accuracy

$$\Omega_{i,\text{wz}} = \frac{3\,(\Psi_{i,\text{wz}+1}-\Psi_{i,\text{wz}})}{r^2\Delta z^2} - \frac{1}{2}\,\Omega_{i,\text{wz}+1}\,,$$

$$\Omega_{\text{w}r,j} = \left[\frac{3\,(\Psi_{\text{w}r+1,j}-\Psi_{\text{w}r,j})}{r^2\Delta r^2} - \frac{1}{2}\,\Omega_{\text{w}r+1,j}\right]\Bigg/\left(1+\frac{3}{2}\frac{\Delta r}{r}\right).$$

Numerical investigations showed that use of the Thom condition leads to results which are in good agreement with the results obtained using the forms of second order of accuracy. On an inclined wall, a value for the vortex was found by interpolation along the normal of the stream function and vorticity at the nearest nodes of the computational grid. Introduction of the model term M_k into the equation of kinetic energy (2) allows one to set the condition on the wall for the dissipation rate of kinetic energy as $\varepsilon_\text{w} = 0$. In the outlet cross section of the considered region, the zero Neumann condition was introduced for all variables. At the inlet boundary, uniform or constant values of the sought variables were assigned.

By virtue of the fact that the convective terms on the solid wall are equal to zero due to the adhesion condition, the value of Ω at the angular point was found directly from the equation of vorticity transfer

$$\frac{\partial}{\partial r}\left[r\,(1+\nu_t)\,\frac{\partial\Omega}{\partial r}\right] + \frac{\partial}{\partial z}\left[r\,(1+\nu_t)\,\frac{\partial\Omega}{\partial z}\right] = -f_\Omega\,.$$

Methods for determination of the vortex at angular points are quite numerous. In [3], seven techniques of formulation of such conditions are given. However, the approaches based on the determination of Ω by arbitrary extrapolation by the values at interior points are incorrect and can lead to instability of solutions. The technique suggested satisfies the equation of vortex transfer at an angular point and has second order of accuracy when central differences are used.

The system of differential equations (1) was replaced by finite-difference analogs and was described by central differences of second order of accuracy. The solution was found by the time-dependent technique directions of variable [4].

Since the problem was solved for rather high Re numbers, to provide stability of the numerical solution and retain second order of accuracy of convective derivatives Shvab et al. [6] used the method of oriented pseudoconvection, which, as an example for the vortex equation, can be represented in the following form:

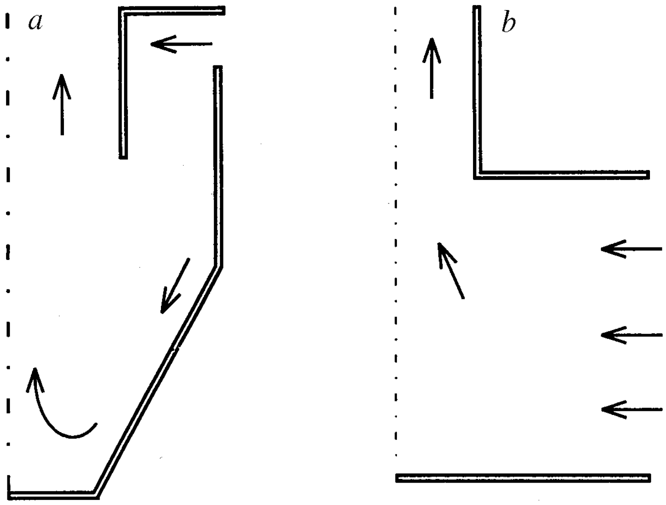


Fig. 1. Geometry of calculated chambers.

on the first time half-step

$$\frac{\mathrm{Re}}{2}\left[r\,\frac{\Omega_{i,j}^{n+1/2}-\Omega_{i,j}^{n}}{\Delta t/2}+\frac{\partial\,(rU_r\Omega)^{n+1/2}}{\partial r}+\frac{\partial\,(rU_z\Omega)^{n}}{\partial z}-\left(2\,\frac{U_\varphi}{r}\,\frac{\partial U_\varphi}{\partial z}\right)^{n}+rD\,(\Omega_{i,j}^{n+1/2}-\Omega_{i,j}^{n})\right]=$$

$$=\frac{\partial}{\partial r}\left[r\,(1+\nu_t)\,\frac{\partial\Omega}{\partial r}\right]^{n+1/2}+\frac{\partial}{\partial z}\left[r\,(1+\nu_t)\,\frac{\partial\Omega}{\partial z}\right]^{n}+f_{\Omega}^{\,n},$$

on the second time half-step

$$\frac{\mathrm{Re}}{2}\left[r\,\frac{\Omega_{i,j}^{n+1}-\Omega_{i,j}^{n+1/2}}{\Delta t/2}+\frac{\partial\,(rU_r\Omega)^{n+1/2}}{\partial r}+\frac{\partial\,(rU_z\Omega)^{n+1}}{\partial z}-\left(2\,\frac{U_\varphi}{r}\,\frac{\partial U_\varphi}{\partial z}\right)^{n}+rD\,(\Omega_{i,j}^{n+1}-\Omega_{i,j}^{n+1/2})\right]=$$

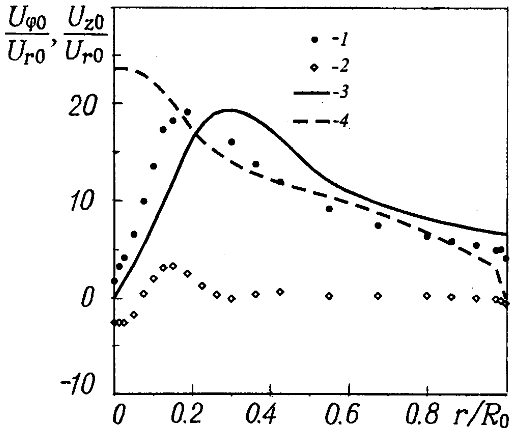
$$=\frac{\partial}{\partial r}\left[r\,(1+\nu_t)\,\frac{\partial\Omega}{\partial r}\right]^{n+1/2}+\frac{\partial}{\partial z}\left[r\,(1+\nu_t)\,\frac{\partial\Omega}{\partial z}\right]^{n+1}+f_{\Omega}^{\,n+1/2},$$

where

$$D=\frac{|U_{ri+1,j}|+|U_{ri-1,j}|}{2\Delta r}+\frac{|U_{zi,j+1}|+|U_{zi,j-1}|}{2\Delta z}.$$

As is shown in [5], on the first iterations the superposition of the difference representation of convection and the introduced additive D is reduced to the known stable counterflow scheme. On the other hand, upon reaching the convergence of the stationary solution, we have $D(\Omega_{i,j}^{n+1}-\Omega_{i,j}^{n}) = 0$.

As an example, Fig. 1 shows the working elements of the cyclone chamber (a) and the vortex chamber with side injection (b) for which numerical calculations were made. To increase the accuracy of the solution near solid surfaces with a limited number of nodes, we can use the analytical transformation of the coordinates of the form $r = z + (-1)^m(1-\alpha)\sin(\pi/z)/(\pi/z)$. As a result, we obtain a bunching difference grid near the inlet boundary and the walls where the gradients of the sought functions are substantial. For $\alpha = 1$ the difference grid is uniform, and for $\alpha = 0$ the grid has the greatest bunching. In the case where m is odd, the bunching occurs only near one boundary; otherwise, the bunching is observed near two boundaries.



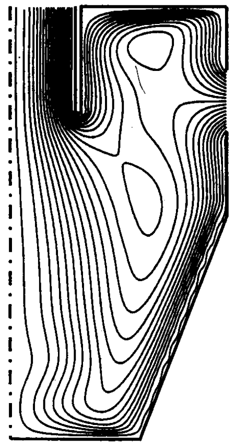


Fig. 2. Profiles of calculated and measured circular and axial velocities in the vortex chamber with side injection (Re = 1470, S = 1.36): 1, 2) U_φ and U_z, experiment [6]; 3, 4) U_φ and U_z, calculation.

Fig. 3. Distribution of streamlines for Re = 10,000 and S = 5.

The reliability of numerical calculations of the hydrodynamics of a twisted turbulent flow within the considered geometries was based on comparison with experimental data. Figure 2 presents calculated and measured [6] profiles of the mean circular and axial velocities in the cross section z/R_0 = 0.9 for the vortex chamber with side injection. The results obtained for U_φ are in rather good agreement with measurements. According to the experimental data, at the outlet boundary one must observe ejection of the gas from outside. However, due to use of the semiempirical k–ε model of turbulence, divergence of the profiles of the axial velocity U_z is observed. To describe this effect more accurately, it is necessary to use adequate models got turbulent processes [7].

As an example, Fig. 3 shows the distribution of stream lines in the cyclone chamber. Large centrifugal forces hold the incoming flow near the inclined side wall, and the descending flow is formed near it. Due to the attenuation of twist near the base, motion of the medium toward the axis and the ascending flow directly to the outlet channel occur. A sharp change in the direction of the flow leads to the formation of the vortex lying near the outlet and the occurrence of the zone of reverse flow near the inclined wall.

The developed method of numerical calculation of twisted turbulent flows is, on the one hand, rather reliable, and, on the other hand, has performed well in optimization of operating-geometric parameters and in development of new pneumatic centrifugal devices.

NOTATION

r, φ, z, radial, angular, and axial coordinates; U_r, U_φ, and U_z, time-averaged velocities along the corresponding coordinates; $\Omega = \frac{1}{r}\left(\frac{\partial U_r}{\partial z} - \frac{\partial U_z}{\partial r}\right)$, vorticity; Ψ, stream function determined as $U_r = \frac{1}{r}\frac{\partial \Psi}{\partial z}$ and $U_z = -\frac{1}{r}\frac{\partial \Psi}{\partial r}$; R_0, radius of the vortex chamber; ν, kinematic viscosity; U_{r0}, mean-flow-rate-radial velocity at the inlet to the chamber; Re = $U_{r0}2R_0/\nu$, Reynolds number; t, time; ν_t, eddy viscosity; k, kinetic energy of turbulence per unit mass; ε, dissipation rate of the kinetic energy of turbulence; f_Ω, right-hand side of the equation of vorticity transfer; G, generation of the energy of pulsatory motion; σ_k, σ_ε, M_k, M_ε, C_μ, $C_{\varepsilon 1}$, and $C_{\varepsilon 2}$, model terms; R_t, turbulent Reynolds number; n, time layer; m, exponent; S, twist parameter of the incoming flow, $S = U_{\varphi 0}/U_{r0}$; D, additive in the method of oriented pseudoconvection; α, coefficient of bunching of the computational grid; Δr and Δz, pitches along the axes; Δt, time step. Subscripts: *wr* and *wz*, refer to the wall; 0, inlet cross section; *i*, numbering of points on the computational grid in the *r* direction; *j*, numbering of points on the computational grid in the *z* direction; *r*, φ, *z*, directionfor the corresponding coordinate directions.

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